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18MAT11

First Semester B.E. Degree Examination, Dec.2019/Jan.2020

Calculus and Linear Algebra

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

Module-1

1 a. With usual notations prove that $\tan\phi = r\left(\dfrac{d\theta}{dr}\right)$. **(06 Marks)**

b. Find the angle between the curves $r = \sin\theta + \cos\theta$ and $r = 2\sin\theta$ **(06 Marks)**

c. Show that the radius of curvature for the catenary of uniform strength $y = a\log\sec\left(\dfrac{x}{a}\right)$ is $a\sec(x/a)$. **(08 Marks)**

OR

2 a. Show that the pairs of curves $r = a(1+\cos\theta)$ and $r = b(1-\cos\theta)$ intersect each other Orthogonally. **(06 Marks)**

b. Find the pedal equation of the curve $r^n = a^n\cos n\theta$. **(06 Marks)**

c. Show that the evolute of $y^2 = 4ax$ is $27ay^2 = 4(x+a)^3$. **(08 Marks)**

Module-2

3 a. Find the Macluarin's series for tanx upto the term x^4. **(06 Marks)**

b. Evaluate $\lim\limits_{x\to 0}\left[\dfrac{a^x+b^x+c^x}{3}\right]^{1/x}$ **(07 Marks)**

c. If $U = f(x-y, y-z, z-x)$, prove that $\dfrac{\partial U}{\partial x}+\dfrac{\partial U}{\partial y}+\dfrac{\partial U}{\partial z}=0$ **(07 Marks)**

OR

4 a. Expand log (sec x) upto the term containing x^4 using Maclaurin's series. **(06 Marks)**

b. Find the extreme values of the function $f(x, y) = x^3 + y^3 - 3x - 12y + 20$. **(07 Marks)**

c. Find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ where $u = x^2+y^2+z^2$, $v = xy+yz+zx$, $w = x+y+z$. **(07 Marks)**

Module-3

5 a. Evaluate $\displaystyle\int_0^1\int_0^{\sqrt{1-x^2}}\int_0^{\sqrt{1-x^2-y^2}} xyz\ dzdydx$ **(06 Marks)**

b. Evaluate $\displaystyle\int_{-2}^{2}\int_0^{\sqrt{4-x^2}}(2-x)dydx$ by changing the order of integration. **(07 Marks)**

c. Prove that $\beta(m,n) = \dfrac{\Gamma(m)\,.\,\Gamma(n)}{\Gamma(m+n)}$ **(07 Marks)**

OR

6 a. Evaluate $\iint y\,dx\,dy$ over the region bounded by the first quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$. **(06 Marks)**

b. Find by double integration the area enclosed by the curve $r = a\,(1 + \text{Cos}\theta)$ between $\theta = 0$ and $\theta = \pi$. **(07 Marks)**

c. Show that $\int_0^{\pi/2} \frac{d\theta}{\sqrt{\text{Sin}\theta}} \times \int_0^{\pi/2} \sqrt{\text{Sin}\theta}\, d\theta = \pi$. **(07 Marks)**

Module-4

7 a. Solve $\frac{dy}{dx} + \frac{y\cos x + \sin y + y}{\sin x + x\cos y + x} = 0$ **(06 Marks)**

b. Solve $r\text{Sin}\theta - \text{Cos}\theta \frac{dr}{d\theta} = r^2$ **(07 Marks)**

c. A series circuit with resistance R, inductance L and electromotive force E is governed by the differential equation $L\frac{di}{dt} + Ri = E$, where L and R are constants and initially the current i is zero. Find the current at any time t. **(07 Marks)**

OR

8 a. Solve $(4xy + 3y^2 - x)dx + x\,(x + 2y)dy = 0$. **(06 Marks)**

b. Find the orthogonal trajectories of the family of parabolas $y^2 = 4ax$. **(07 Marks)**

c. Solve $p^2 + 2py\cot x = y^2$. **(07 Marks)**

Module-5

9 a. Find the rank of $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ by elementary row transformations. **(06 Marks)**

b. Apply Gauss-Jordan method to solve the system of equations

$2x_1 + x_2 + 3x_3 = 1,$
$4x_1 + 4x_2 + 7x_3 = 1,$
$2x_1 + 5x_2 + 9x_3 = 3.$ **(07 Marks)**

c. Find the largest Eigen value and the corresponding Eigen vector of the matrix $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$ by power method. Using initial vector $(100)^T$. **(07 Marks)**

OR

10 a. Solve by Gauss elimination method

$x - 2y + 3z = 2,$
$3x - y + 4z = 4,$
$2x + y - 2z = 5$ **(06 Marks)**

b. Solve the system of equations by Gauss-Seidal method

$20x + y - 2z = 17,$
$3x + 20y - z = -18,$
$2x - 3y + 20z = 25$ **(07 Marks)**

c. Reduce the matrix $A = \begin{bmatrix} -1 & 3 \\ -2 & 4 \end{bmatrix}$ to the diagonal form. **(07 Marks)**

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